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Tencent and Facebook Data Validate Metcalfe's Law

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Abstract In 1980s, Robert Metcalfe, the inventor of Ethernet, proposed a formulation of network value in terms of the network size (the number of nodes of the network), which was later named as Metcalfe's law. The law states that the value V of a network is proportional to the square of the size n of the network, i.e., $V \propto n^2$. Metcalfe's law has been influential and an embodiment of the network effect concept. It also generated many controversies. Some scholars went so far as to state "Metcalfe's law is wrong" and "dangerous". Some other laws have been proposed, including Sarnoff's law ($V \propto n$), Odlyzko's law ($V \propto n\log(n)$), and Reed's law ($V \propto 2^n$). Despite these arguments, for 30 years, no evidence based on real data was available for or against Metcalfe's law. The situation was changed in late 2013, when Metcalfe himself used Facebook's data over the past 10 years to show a good fit for Metcalfe's law. In this paper, we expand Metcalfe's results by utilizing the actual data of Tencent (China's largest social network company) and Facebook (the world's largest social network company). Our results show that: 1) of the four laws of network effect, Metcalfe's law by far fits the actual data the best; 2) both Tencent and Facebook data fit Metcalfe's law quite well; 3) the costs of Tencent and Facebook are proportional to the squares of their network sizes, not linear; and 4) the growth trends of Tencent and Facebook monthly active users fit the netoid function well.

Keywords network effect, Metcalfe's law, cost, netoid function

1 Introduction

Network effect has become an influential concept not only in the technology field, but also in economy and business, social sciences, and even global public events[1-2]. A network effect is the effect that a network's *value* V is dependent on its *size* n (the number of its nodes)[3]. Four laws have been proposed to provide more precise definitions and characterizations of network effect. They are

- Sarnoff's law[3]: $V \propto n$,
- Odlyzko's law[4]: $V \propto n\log(n)$,
- Metcalfe's law[5]: $V \propto n^2$, and
- Reed's law[6]: $V \propto 2^n$.

Many papers are published[3-9] arguing for or against these laws. However, no actual evidence was available in the literature to validate these laws with real data until December 2013, when Robert Metcalfe himself utilized Facebook's actual data over the past decade to show a good fit to Metcalfe's law[8].

There are four key points in Metcalfe's experiments: 1) Metcalfe reiterated the hypotheses proposed 40 years ago, i.e., a network has a value of $V \propto n^2$ but a cost of $C \propto n$; 2) Facebook's network size n is defined as the number of its monthly active users (MAUs), while Facebook's network value V is defined as its revenue (as a proxy); 3) the Facebook data indeed fit Metcalfe's law well, i.e., Facebook's revenue is proportional to the square of the number of its MAUs; 4) a function, called *netoid* function, is defined to describe the growth trend of a network.

Several key questions are not answered by Metcalfe's paper.

- Is Metcalfe's law only valid for Facebook, a company in a developed country serving worldwide users?

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This paper provides additional evidence by using real data from Tencent, a company in a developing country mostly serving Chinese users.

• Which of the four laws best fits real data?

This paper utilizes the actual data of Tencent and Facebook to validate the four laws, and shows that Metcalfe's law fits the best.

• Is Metcalfe's linear-cost hypothesis ($C \propto n$) valid?

We show that it does not hold for the Tencent and the Facebook data.

2 Material and Method

2.1 Data Sources

To validate the four network effect laws with the real data of Tencent and Facebook network, we need actual data of more than a decade for the network value V and the network size n. In addition, we need actual data for the network cost C to validate Metcalfe's linear-cost hypothesis. Since Facebook and Tencent are both public companies, all actual data are available from their prospectus and financial reports①~③, and summarized in Appendix A1.

We follow Metcalfe's methodology to define network size, value, and cost[8]. We use the revenues as proxies for Tencent's and Facebook's network values. We define cost as the total business cost (tax included) incurred in generating revenue. In other words, the cost is the revenue minus the net profit.

We use the number of MAUs to represent the network size (number of nodes) of Tencent and Facebook. MAU is a metric to count the number of unique users who use the social networking services over the past 30 days. Facebook's MAUs numbers are published in its financial reports. Tencent's MAUs numbers are defined as the sum of QQ MAUs and Weixin (WeChat) MAUs, as all the 250 Tencent services use these two user account systems.

2.2 Value, Cost, and Trend Functions

Variable definitions are listed in Table 1. The formulations of the value, cost, and trend functions are listed in Table 2. To maintain a simple and easy-to-use formulation, we only consider the major term, ignoring secondary terms.

Table 1. Variable Definitions

Symbol	Unit	Definition	Data Source
V	USD	Value of a network	Revenue
C	USD	Cost of a network	Revenue – net profit
n	MAU	Number of nodes of a network	MAU
netoid	MAU	Growth trend of size n	MAU

Table 2. Models of Network Laws, Cost Function, and Netoid Function

	Model	Unit of Parameters
Sarnoff's function	$V = a \times n$	a: USD/MAU
Reed's function	$V = a \times (2^n - 1)$	a: USD/MAU
Odlyzko's function	$V = a \times n \log_2(n)$	a: USD/MAU
Metcalfe's function	$V = a \times n^2$	a: USD/MAU^2
Cost function	$C = a \times n^2$	a: USD/MAU^2
Netoid function	$Netoid = p/(1 + e^{-v \times (t-h)})$	p: MAU, h: year, v: year^{-1}

Formulating the value functions for the four network effect laws is straightforward, as specified in Section 1. The proportionality constant of the four functions, a, is named as Sarnoff's coefficient, Odlyzko's coefficient, Metcalfe's coefficient, and Reed's coefficient, respectively. If a network has a larger a than another network, the former network provides a larger value per user (per node) than the latter network. When the number of users of a network is 0, the value of the network should be 0. Thus we use $2^n - 1$, not 2^n, in Reed's function, to ensure the curve of Reed's law can pass through the origin.

Formulating the cost function is not so straightforward. Metcalfe hypothesized that the cost of a network is proportional to its size. But this linear-cost hypothesis deviates too much from the real data of both Facebook and Tencent, and we have to abandon it and try other formulations. It turns out that a quadratic-cost hypothesis fits Tencent and Facebook data much better. Thus a cost function is used whereby the cost is proportional to the square of the network size, i.e., $C = a \times n^2$.

We use Metcalfe's netoid function[8] to represent the growth trend of the network size n with respect to time t.

$$Netoid = p/(1 + e^{-v \times (t-h)}).$$

① Tencent financial reports. http://www.tencent.com/en-us/ir/reports.shtml, Mar. 2015.

② http://www.sec.gov/Archives/edgar/data/1326801/000119312512034517/d287954ds1.htm#toc, Feb. 2012.

③ Facebook financial reports. http://investor.fb.com/, Feb. 2015.

The three parameters p, v, h have the following meanings:

- p: the peak value representing the maximum value of the number of MAUs;
- v: the virality or speed with which adoption occurs;
- h: the point in time at which the growth rate is maximum, when the network size reaches half the peak.

2.3 Curve Fitting Method

When validating his law, Metcalfe "fiddled with the slider parameter" provided by the Python programming language to achieve a good visual fit to the actual data[8]. Although this method is intuitive and of great convenience, it is not so accurate and may miss some important details. We use the least squares method in curve fitting to fit Tencent and Facebook data to the value, cost, and trend functions. In particular, we use the least squares function "leastsq" provided by SciPy, an open source Python library.

3 Results and Discussions

3.1 Value Functions

Table 3 shows the fitting results and corresponding root-mean-square deviations (RMSDs) of the four network effect functions, for Tencent data and Facebook data. The corresponding fitting curves graphics are shown in Fig.1 and Fig.2 respectively. The contrast of the actual data and the derived values of the value functions for Tencent and Facebook data is shown in Appendix A2 and Appendix A3 respectively.

The fitting RMSDs of Metcalfe's functions of Tencent and Facebook are significantly smaller than those of the other network functions. For Tencent, the ratios of the RMSD of Metcalfe's function to the RMSDs of Sarnoff's function, Odlyzko's function, and Reed's function are about 1/13, 1/13, and 1/45 respectively. For Facebook, the corresponding ratios are about 1/2, 1/2, and 1/8 respectively. These results show that for both Tencent and Facebook data, Metcalfe's function not only fits the real data model well, but also is far more accurately than the other three laws.

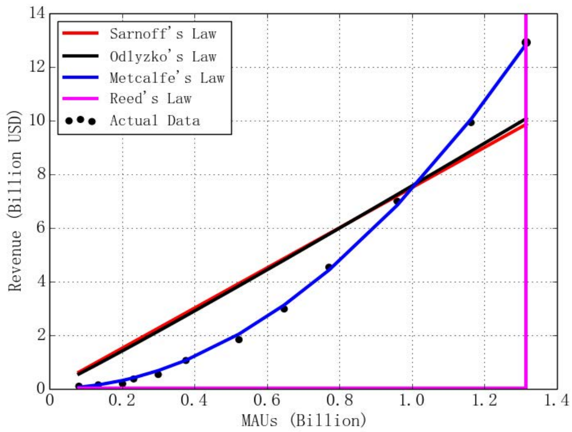

Fig.1. Value curves of Tencent.

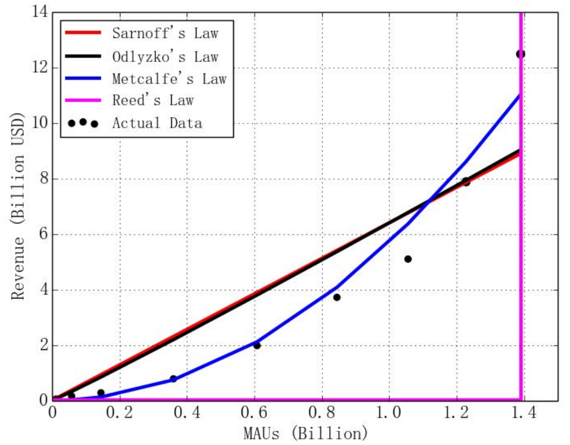

Fig.2. Value curves of Facebook.

3.2 Cost Functions

Fig.3 and Fig.4 show the fitting curves of the cost functions of Tencent and Facebook respectively. The cost functions are $C_{\text{Tencent}} = 5.22 \times 10^{-9} \times n^2$ and $C_{\text{Facebook}} = 4.56 \times 10^{-9} \times n^2$. The RMSDs are 0.22 (billion USD) and 0.39 (billion USD), respectively. Thus

Table 3. Fitting Results of the Four Network Effect Laws

	Tencent Data		Facebook Data	
	Value Functions	RMSDs	Value Functions	RMSDs
Sarnoff's function	$V_{\text{Tencent}} = 7.49n$	1.61	$V_{\text{Facebook}} = 6.39n$	1.51
Odlyzko's function	$V_{\text{Tencent}} = 0.25 \times n \log_2(n)$	1.51	$V_{\text{Facebook}} = 0.21 \times n \log_2(n)$	1.45
Metcalfe's function	$V_{\text{Tencent}} = 7.42 \times 10^{-9} \times n^2$	0.12	$V_{\text{Facebook}} = 5.70 \times 10^{-9} \times n^2$	0.64
Reed's function	$V_{\text{Tencent}} = 2^{-1.32\times10^9} \times (2^n - 1)$	5.38	$V_{\text{Facebook}} = 2^{-1.39\times10^9} \times (2^n - 1)$	4.88

Note: V: USD, n: MAU, RMSD: billion USD.

based on the actual data of Tencent and Facebook, the assumption that the cost of a network company is proportional to the square of the number of its MAUs is correct.

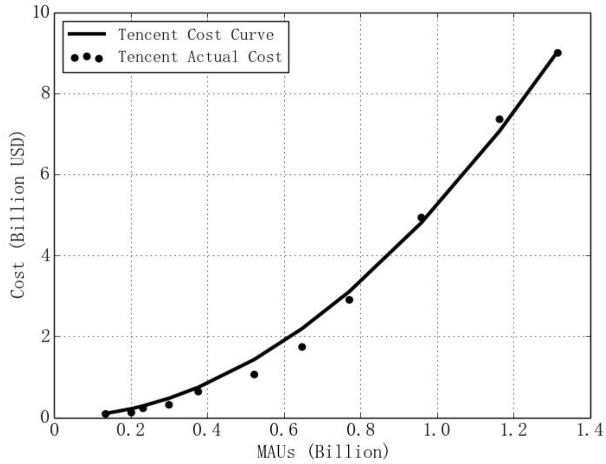

Fig.3. Cost curves of Tencent.

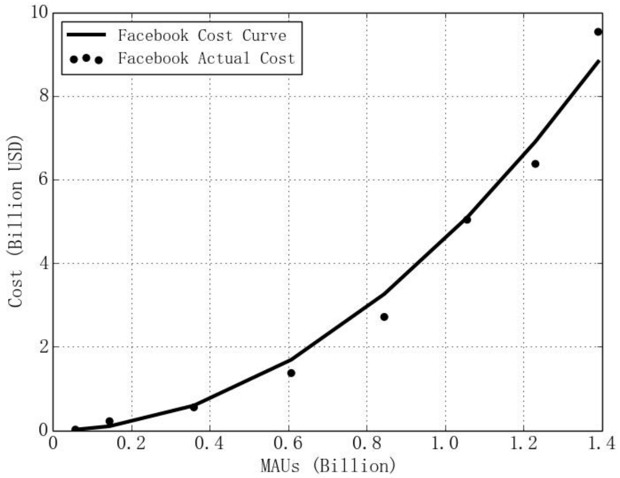

Fig.4. Cost curves of Facebook.

3.3 Trend Functions

Fig.5 and Fig.6 present the growth trend of MAUs of Tencent and Facebook respectively. The netoid functions are as follows:

$$Netoid_{\text{Tencent}} = 2.41 \times 10^9/(1 + e^{-0.30\times(t-2\,013.35)}),$$
$$Netoid_{\text{Facebook}} = 1.45 \times 10^9/(1 + e^{-0.77\times(t-2\,010.56)}).$$

The RMSDs of Tencent and Facebook are 0.015 (billion USD) and 0.028 (billion USD), respectively. We compare the values of the two companies' MAUs derived from the netoid functions with the actual data (as shown in Appendix A4) to validate the netoid functions.

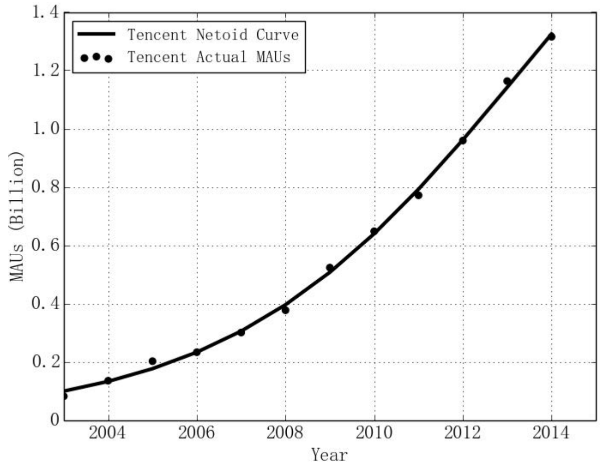

Fig.5. Netoid curves of Tencent.

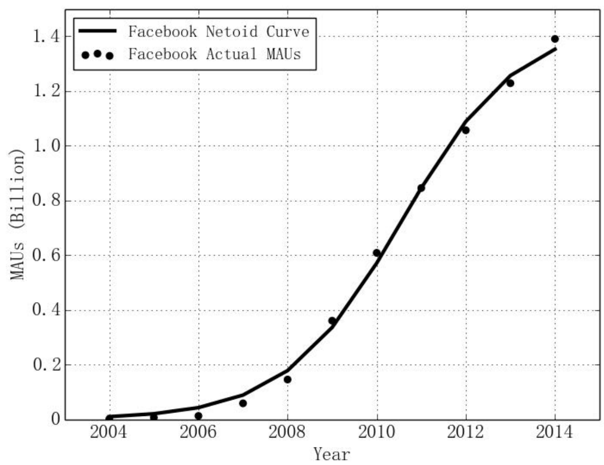

Fig.6. Netoid curves of Facebook.

4 Related Work

Metcalfe's law was proposed in the early 1980s, which states that the value of a network is proportional to the square of the size of the network.

Recently, a few papers have appeared to argue for or against Metcalfe's law. Odlyzko *et al.* described Metcalfe's law as both "wrong" and "dangerous"[4]. They argued that if Metcalfe's law is true, then two networks ought to interconnect regardless of their relative sizes. They proposed Odlyzko's law which states that the value of a network grows in proportion to $n\log(n)$. Van Hove proposed that the inference of Odlyzko is flawed. He argued that Metcalfe's law is not so wrong after all[9].

However, all of these arguments do not have evidence that is based on real data. Madureira *et al.* exploited the Eurostat data to validate Metcalfe's law and concluded that the value of a network can have either a quadratic or a linear dependency with the size of the network[7]. In late 2013, Metcalfe used Facebook's data over the past 10 years to show a good fit for Metcalfe's law[8].

5 Conclusions

Tencent and Facebook have big differences in revenue, cost, business model, and technology. Yet both of their actual data fit Metcalfe's law well. The Metcalfe's functions of them are $V_{\text{Tencent}} = 7.42 \times 10^{-9} \times n^2$ and $V_{\text{Facebook}} = 5.70 \times 10^{-9} \times n^2$ respectively.

The relationships between the costs of Tencent and Facebook and their network size are quadratic, rather than linear. The cost functions of them are $C_{\text{Tencent}} = 5.22 \times 10^{-9} \times n^2$ and $C_{\text{Facebook}} = 4.56 \times 10^{-9} \times n^2$, respectively.

The growth trend of MAUs of Tencent and Facebook over the past decade can be modeled by the netoid functions. The netoid functions are $Netoid_{\text{Tencent}} = 2.41 \times 10^9/(1 + e^{-0.30\times(t-2013.35)})$ and $Netoid_{\text{Facebook}} = 1.45 \times 10^9/(1 + e^{-0.77\times(t-2\,010.56)})$, respectively.

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Appendix A1 Actual Data of Tencent and Facebook

Year	Tencent Data			Facebook Data		
	MAUs (Billion)	Revenues (Billion USD)	Cost (Billion USD)	MAUs (Billion)	Revenues (Billion USD)	Cost (Billion USD)
2003	0.081 5	0.088 7	0.049 8	N/A	N/A	N/A
2004	0.134 8	0.138 1	0.084 2	0.001	0.000 382	N/A
2005	0.201 9	0.176 8	0.116 7	0.006	0.009 000	N/A
2006	0.232 6	0.358 6	0.222 4	0.012	0.048 000	N/A
2007	0.300 2	0.523 1	0.308 4	0.058	0.153 000	0.015
2008	0.376 6	1.047 0	0.634 8	0.145	0.272 000	0.216
2009	0.522 9	1.822 0	1.057 2	0.360	0.777 000	0.548
2010	0.647 6	2.967 0	1.741 1	0.608	1.974 000	1.368
2011	0.771 0	4.523 0	2.899 7	0.845	3.711 000	2.711
2012	0.959 0	6.983 0	4.949 3	1.060	5.089 000	5.036
2013	1.163 0	9.913 0	7.360 4	1.230	7.872 000	6.372
2014	1.315 0	12.899 0	8.995 0	1.390	12.470 000	9.530

Appendix A2 Actual Data Versus Derived Values of the Value Functions of Tencent

Year	Actual Revenues (Billion USD)	Sarnoff's Function (Billion USD)	Odlyzko's Function (Billion USD)	Metcalfe's Function (Billion USD)	Reed's Function (Billion USD)
2003	0.088 7	0.610 4	0.535 5	0.049 3	$2^{-1.234\times10^9}$
2004	0.138 1	1.009 7	0.910 1	0.134 8	$2^{-1.180\times10^9}$
2005	0.176 8	1.511 9	1.392 2	0.302 3	$2^{-1.113\times10^9}$
2006	0.358 6	1.742 1	1.616 2	0.401 4	$2^{-1.082\times10^9}$
2007	0.523 1	2.248 5	2.113 5	0.668 7	$2^{-1.015\times10^9}$
2008	1.047 0	2.820 7	2.682 2	1.052 4	$2^{-0.938\times10^9}$
2009	1.822 0	3.916 5	3.786 1	2.028 8	$2^{-0.792\times10^9}$
2010	2.967 0	4.850 5	4.738 9	3.111 8	$2^{-0.667\times10^9}$
2011	4.523 0	5.774 8	5.690 4	4.410 8	$2^{-0.554\times10^9}$
2012	6.983 0	7.182 9	7.153 4	6.824 0	$2^{-0.356\times10^9}$
2013	9.913 0	8.710 8	8.756 0	10.036 1	$2^{-0.152\times10^9}$
2014	12.899 0	9.849 3	9.958 6	12.830 8	2^0

Appendix A3 Actual Data Versus Derived Values of the Value Functions of Facebook

Year	Actual Revenues (Billion USD)	Sarnoff's Function (Billion USD)	Odlyzko's Function (Billion USD)	Metcalfe's Function (Billion USD)	Reed's Function (Billion USD)
2004	0.000 382	0.006 39	0.004 185 629	0.000 005 7	$2^{-1.389\times10^9}$
2005	0.009 000	0.038 34	0.028 370 829	0.000 205 2	$2^{-1.384\times10^9}$
2006	0.048 000	0.076 68	0.059 261 658	0.000 820 8	$2^{-1.378\times10^9}$
2007	0.153 000	0.370 62	0.314 116 714	0.019 174 8	$2^{-1.332\times10^9}$
2008	0.272 000	0.926 55	0.825 544 495	0.119 842 5	$2^{-1.245\times10^9}$
2009	0.777 000	2.300 40	2.148 810 678	0.738 720 0	$2^{-1.030\times10^9}$
2010	1.974 000	3.885 12	3.725 638 060	2.107 084 8	$2^{-0.782\times10^9}$
2011	3.711 000	5.399 55	5.262 169 039	4.069 942 5	$2^{-0.545\times10^9}$
2012	5.089 000	6.747 84	6.647 469 486	6.356 275 2	$2^{-0.334\times10^9}$
2013	7.872 000	7.846 92	7.786 341 921	8.595 508 8	$2^{-0.160\times10^9}$
2014	12.470 000	8.882 10	8.865 714 575	11.012 970 0	2^0

Appendix A4 Actual Data Versus Derived Values of the Netoid Functions

Year	Tencent Data		Facebook Data	
	MAUs (Billion)	Values of Netoid Function (Billion)	MAUs (Billion)	Values of Netoid Function (Billion)
2003	0.081 5	0.101 9	N/A	N/A
2004	0.134 8	0.135 6	0.001	0.009 223 343
2005	0.201 9	0.179 5	0.006	0.019 774 386
2006	0.232 6	0.236 1	0.012	0.042 043 085
2007	0.300 2	0.308 1	0.058	0.087 849 076
2008	0.376 6	0.398 1	0.145	0.177 277 077
2009	0.522 9	0.508 1	0.360	0.335 329 627
2010	0.647 6	0.638 7	0.608	0.571 067 738
2011	0.771 0	0.789 0	0.845	0.846 653 549
2012	0.959 0	0.955 6	1.056	1.090 262 724
2013	1.163 0	1.132 8	1.228	1.257 836 185
2014	1.315 0	1.313 2	1.390	1.354 208 636