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Facebook and Tencent Data Fit a Cube Law Better than Metcalfe's Law

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Abstract Metcalfe's law states that the value of a network grows as the square of the number of its users ($V \propto n^2$), which was validated by actual data of Facebook and Tencent in 2013–2015. Since then, the users and the values of Facebook and Tencent have increased significantly. Is Metcalfe's law still valid? This paper leverages the latest data of Facebook and Tencent to fit the network effect laws and makes the following observations: 1) actual data of network values fit a cube law ($V \propto n^3$) better than Metcalfe's law; 2) actual data of network costs fit a cube law; 3) actual data of network sizes show a growth trend matching the netoid function well. We also discuss the underlying factors affecting such observations and the generality of the network effect laws.

Keywords network effect, Metcalfe's law, cube law, netoid function

1 Introduction

Metcalfe's law was proposed in 1980s by Robert Metcalfe, the inventor of Ethernet, which states that the value of a network grows as the square of the number of its users[1]. Metcalfe's law provides a precise definition of network effect and has played an important role in many fields, including computer science, economics, and social sciences. Researches were published to discuss the validity of Metcalfe's law or to validate the law by fitting actual data[2–6]. For example, in 2013, Metcalfe himself utilized Facebook's actual data over 2004–2013 to show a good fit to Metcalfe's law[5] by assuming the network size as the number of monthly active users (MAUs) and the network value as the company's annual revenue. In 2015, Zhang *et al.* expanded Metcalfe's results by utilizing the actual data of Tencent and Facebook and found that of four network effect laws, Metcalfe's law fits the actual data the best[6].

Since then, the network sizes and external environments of Facebook and Tencent have undergone tremendous changes. The technology architectures of both companies are also evolving to meet business needs[7, 8]. The number of Facebook's MAUs increased from 1.23 billion in 2013 to 2.80 billion in 2020, and its revenue increased from $7.87 billion in 2013 to $85.97 billion in 2020①. Tencent's MAUs also grew significantly, from 1.16 billion in 2013 to 1.89 billion in 2019. Its revenue increased from $9.91 billion in 2013 to $54.08 billion in 2019②.

Is Metcalfe's law still valid for large social networks? This paper uses the latest public data of Facebook and Tencent to validate the network effect laws. The actual data of the two networks are also used to find the best fitting cost functions and trend functions.

Although Facebook and Tencent have the commonality of being a large social network, the two networks are quite different. The diversity may help

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①Facebook financial reports. http://investor.fb.com/, Nov. 2022.
②Tencent financial reports. http://www.tencent.com/en-us/ir/reports.shtml, Nov. 2022.

demonstrate the universality of the network effect laws. The differences are reflected in four aspects.

1) *Different User Distributions*. Users from US & Canada, Europe, and Asia-Pacific account for 9.22%, 14.98%, and 42.87% of Facebook's MAUs in 2020 respectively, while the vast majority of Tencent MAUs are from China.

2) *Different Revenue Composition*. Up to 97.90% of Facebook's revenue comes from advertising in 2020 while advertising only accounts for 18.12% of Tencent's revenue in 2019.

3) *Different Product Forms*. Facebook's social network is composed of multiple feature-specific APPs, including Facebook, Messenger, WhatsApp, Instagram, and so on. In contrast, Tencent's social network is built based on two feature-rich super APPs: QQ and WeChat.

4) *Different Network Openness*. Facebook allows users to view their friends' friends lists and visit their posts, while Tencent's users can only connect to their own friends, that is, "friends of friends are not my friends".

2 Related Work

Metcalfe's law has been discussed for more than 40 years since it was proposed. Some scholars state that the square relationship in Metcalfe's law either overestimates or underestimates the value of the network. Several competing laws have been proposed to explore the relationship between network value V and the number of users n, such as Sarnoff's law[2] ($V \propto n$), Odlyzko's law[3] ($V \propto n\log_2 n$), and Reed's law[9] ($V \propto 2^n$).

Metcalfe's law has also been validated with different data sources in multiple fields. The most representative work is that Metcalfe successfully fitted his law to Facebook's annual revenues in 2013[5]. In 2015, Zhang *et al.*[6] utilized data from Facebook and Tencent to verify Sarnoff's law, Odlyzko's law, Metcalfe's law, and Reed's law. It found that Metcalfe's law fits the actual data the best[6]. Van Hove extended Zhang *et al.*'s approach by explicitly controlling changes in network quality (cost per node) over time and filtered out revenues and costs that are unrelated to Tencent's core services (social network)[10]. In 2016, Van Hove presented a test of Metcalfe's law and pointed out several difficulties encountered when testing the network laws, including the scope of the law, the boundaries of the market, the network size, and so on[11]. Van Hove's experimental results proved that Metcalfe's law outperforms the other laws even more clearly[10, 11].

In the field of digital currency, Alabi proposed that the digital blockchain network appears to be following Metcalfe's law[12] and Peterson demonstrated that bitcoin's medium- to long-term price follows Metcalfe's law[13]. In the field of Industrial Internet, Liu proposed that all networked people, machines and things have the possibility of collaboration and interaction, so can they create value beyond Metcalfe's law[14]? In the field of scientific enterprise, Weis and Jacobson hypothesized that as the number of connections between research projects increases, the value created by the scientific research transitions from linear to geometric in the number of funded projects[15].

3 Material and Method

3.1 Definitions of Terms

As is shown in Table 1, this paper follows Metcalfe's methodology to define the value, cost, and size of the network. The revenue, cost, and MAUs data of Facebook and Tencent are published in their financial reports. We organize and publish them publicly as a supplementary material online③, which are labeled as Appendix A1. Besides, some of the experimental data in this paper are included in the supplementary material and are cited in the form of Appendix A2–Appendix A9.

Table 1. Definitions of Terms

Symbol	Unit	Definition	Term in Financial Reports
V	USD	Value of a network	Revenue
C	USD	Cost of a network	Revenue–net profit
n	MAU	Network size, i.e., the number of MAUs	MAU
netoid	MAU	Growth trend of size n	MAU

The value V is defined as the revenue and the cost C is defined as the total business cost (tax included) incurred in generating revenue. That is, cost is revenue minus net profit.

The network size n is defined as the number of monthly active users (MAUs). Facebook's MAUs nu-

③https://github.com/zxzStar/Network-effect-laws, Nov. 2022.

mbers are from 2004 to 2020, including users of Facebook and Messenger, which is consistent with Facebook's annual report definition. Tencent's MAUs numbers are defined as the sum of QQ MAUs and WeChat MAUs, as all Tencent's services are based on these two user account systems. Currently, Tencent has not released the total MAUs of QQ in 2020. Therefore, we use the Tencent MAUs data from 2003 to 2019.

3.2 Network Value, Cost, and Trend Functions

To facilitate the data fitting, this paper formalizes the functions of the network value, cost, and MAUs' growth trend, which are listed in Table 2.

Table 2. Network Value, Cost, and Trend Functions

	Function	Model	Unit of Parameters
Value functions	Sarnoff's function[2]	$V = a \times n$	a: USD/MAU
	Odlyzko's function[3]	$V = a \times n\log_2 n$	a: USD/MAU
	Metcalfe's function[1]	$V = a \times n^2$	a: USD/MAU^2
	Cube function	$V = a \times n^3$	a: USD/MAU^3
	Quad function	$V = a \times n^4$	a: USD/MAU^4
	Reed's function[9]	$V = a \times 2^n$	a: USD/MAU
Cost functions	Cost Square function	$C = b \times n^2$	b: USD/MAU^2
	Cost Cube function	$C = b \times n^3$	b: USD/MAU^3
	Cost Quad function	$C = b \times n^4$	b: USD/MAU^4
Trend function	Netoid function	$netoid = \frac{p}{1+\mathrm{e}^{-v\times(t-h)}}$	p: MAU, h: year, v: year^{-1}

For the network value, this paper formalizes four value functions according to the description of the network effect laws, including Sarnoff's function, Odlyzko's function, Metcalfe's function, and Reed's function. In addition, this paper defines the Cube law and the Quad law, and formalizes the Cube function and the Quad function.

Definition 1 (Cube Law: $V \propto n^3$). *The value of a network is proportional to the cube of the number of its users.*

Definition 2 (Quad Law: $V \propto n^4$). *The value of a network is proportional to the quad (fourth power) of the number of its users.*

Inspired by the network effect laws, this paper defines three network cost functions to explore the relationship between the network cost and its size. The cost functions are the Cost Square function, the Cost Cube function, and the Cost Quad function.

This paper uses Metcalfe's netoid function[5] as the trend function to show the growth trend of the network size with respect to time t. Here, p refers to the peak value of the network size, v refers to the adoption speed, and h refers to the time point at which the growth rate is maximum, that is, the network size *netoid*(t) reaches half the peak.

3.3 The Curve Fitting Method

This paper uses the "least squares" method to fit Facebook and Tencent data to the value, cost, and trend functions. The "least squares" method is a statistical procedure to find the best fit for a set of data points by minimizing the sum of the squares of the errors made in the results from the plotted curve. In practice, we use the least squares function "leastsq" provided by SciPy④, an open-source Python library. Root-mean-square errors (RMSEs) are used to show the fitting errors between the functions and the real data. The units of RMSEs are billion USD. The normalized root-mean-square errors (NRMSEs) are defined as the ratio of the RMSEs to the difference between the maximum and the minimum of the actual data, i.e., *max* and *min*, to represent normalized errors in percentage. That is,

$$NRMSE = RMSE/(max - min).$$

To explore the change of network value with the expansion of the network size, this paper fits the real data to the value functions from the beginning year to every of the next 16 years.

4 Results and Discussions

4.1 Value Functions

This paper leverages the actual data of Facebook and Tencent to validate the network value functions. The fitting results are shown in Table 3. By comparing the NRMSEs, we find that the best-fitting rela-

④https://scipy.org/, Mar. 2023.

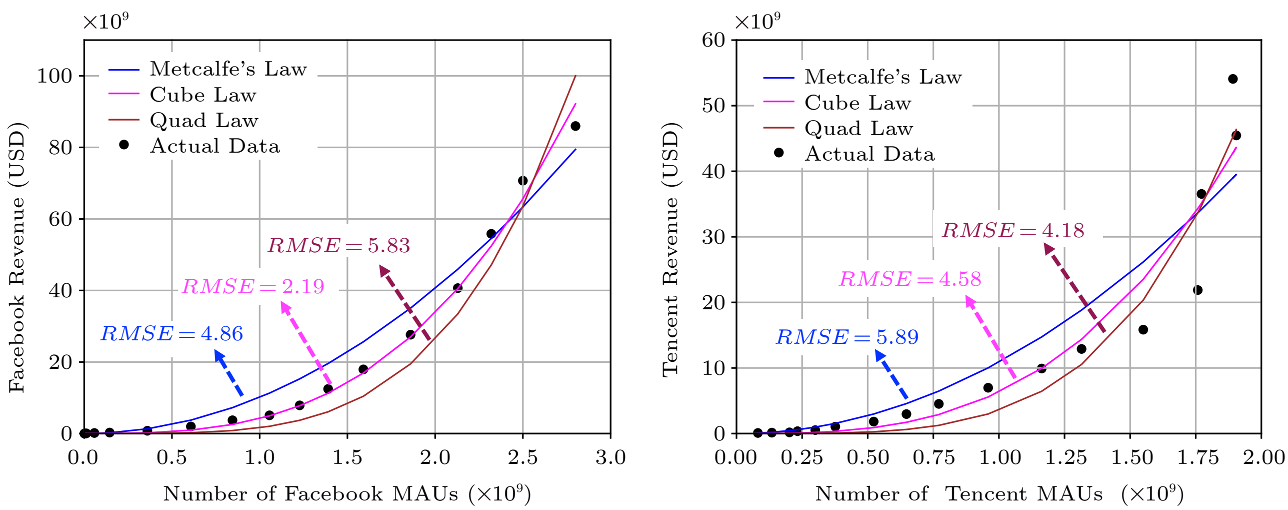
Table 3. Fitting Results of Network Value Functions

	Facebook Data (2004–2020)		Tencent Data (2003–2019)	
	Value Function	NRMSE (%)	Value Function	NRMSE (%)
Sarnoff's function	$V_{\text{Facebook}} = 20.97 \times n$	14.84	$V_{\text{Tencent}} = 16.71 \times n$	16.85
Odlyzko's function	$V_{\text{Facebook}} = 0.98 \times n\log_2 n$	14.32	$V_{\text{Tencent}} = 0.55 \times n\log_2 n$	16.43
Metcalfe's function	$V_{\text{Facebook}} = 10.13 \times 10^{-9} \times n^2$	5.65	$V_{\text{Tencent}} = 10.89 \times 10^{-9} \times n^2$	10.91
Cube function	$V_{\text{Facebook}} = 4.20 \times 10^{-18} \times n^3$	**2.55**	$V_{\text{Tencent}} = 4.58 \times 10^{-18} \times n^3$	8.48
Quad function	$V_{\text{Facebook}} = 1.63 \times 10^{-27} \times n^4$	6.79	$V_{\text{Tencent}} = 3.52 \times 10^{-27} \times n^4$	**7.73**

tionship between the network value and network size has exceeded the square law, reaching the Cube law and even approaching the Quad law.

As is shown in Fig.1, Metcalfe's law can reflect the growing trend of network value. However, the fitting results of the Cube law are better than those of Metcalfe's law, and the NRMSEs are the smallest. For Tencent, the NRMSE of the Quad law is 7.73%, which is smaller than that of the Cube law (8.48%) and Metcalfe's law (10.91%).

Table 4 and Table 5 show the coefficients (a in the functions), RMSEs, and NRMSEs of Facebook and Tencent for various time periods with the same start year and different end years respectively. Due to the space limit, the unit of RMSE is omitted in the two tables. We assume at least six years for a period. The bold numbers in the tables represent the smallest fitting errors among the network value functions



(a) (b)

Fig.1. Value curves of (a) Facebook and (b) Tencent.

Table 4. Facebook's Coefficients (a), RMSEs, and NRMSEs of the Network Value Functions

Start Year	End Year	Sarnoff's Function a	RMSE	NRMSE (%)	Odlyzko's Function a	RMSE	NRMSE (%)	Metcalfe's Function $a \times 10^{-9}$	RMSE	NRMSE (%)	Cube Function $a \times 10^{-18}$	RMSE	NRMSE (%)	Quad Function $a \times 10^{-27}$	RMSE	NRMSE (%)
2004	2010	2.92	0.14	7.12	0.15	0.13	6.49	**5.44**	**0.0861**	**4.36**	9.12	0.17	8.86	14.93	0.23	11.77
2004	2011	3.77	0.31	8.49	0.19	0.29	7.86	**5.25**	**0.0856**	**2.31**	6.53	0.28	7.51	7.80	0.42	11.32
2004	2012	4.27	0.40	7.86	0.21	0.37	7.19	**4.80**	**0.1700**	**3.37**	4.83	0.49	9.67	4.66	0.72	14.13
2004	2013	5.10	0.75	9.56	0.25	0.71	9.00	**5.03**	**0.2100**	**2.68**	4.45	0.51	6.45	3.77	0.81	10.31
2004	2014	6.39	1.51	12.07	0.31	1.45	11.61	5.70	0.6400	5.10	**4.56**	**0.49**	**3.96**	3.49	0.82	6.61
2004	2015	7.88	2.36	13.16	0.38	2.27	12.69	6.32	0.9700	5.39	**4.50**	**0.48**	**2.68**	3.03	1.08	6.03
2004	2016	9.93	3.78	13.68	0.47	3.65	13.22	7.08	1.5000	5.44	**4.38**	**0.52**	**1.87**	2.53	1.68	6.06
2004	2017	12.47	5.74	14.12	0.59	5.55	13.64	7.91	2.2400	5.52	**4.28**	**0.57**	**1.41**	2.16	2.36	5.81
2004	2018	15.35	8.19	14.66	0.73	7.92	14.19	8.85	3.4600	6.19	**4.38**	**0.70**	**1.26**	2.03	2.54	4.56
2004	2019	18.24	10.66	15.07	0.86	10.31	14.59	9.69	4.5800	6.48	**4.44**	**0.81**	**1.14**	1.92	2.87	4.06
2004	2020	20.97	12.76	14.84	0.98	12.31	14.32	10.13	4.8600	5.65	**4.20**	**2.19**	**2.55**	1.63	5.83	6.79

Table 5. Tencent's Coefficients (a), RMSEs, and NRMSEs of the Network Value Functions

Start Year	End Year	Sarnoff's Function			Odlyzko's Function			Metcalfe's Function			Cube Function			Quad Function		
		a	RMSE	NRMSE (%)	a	RMSE	NRMSE (%)	$a\times 10^{-9}$	RMSE	NRMSE (%)	$a\times 10^{-18}$	RMSE	NRMSE (%)	$a\times 10^{-27}$	RMSE	NRMSE (%)
2003	2010	3.42	0.43	14.91	0.12	0.40	14.07	**6.93**	**0.07**	**2.35**	11.65	0.21	7.34	18.50	0.36	12.34
2003	2011	4.31	0.64	14.54	0.15	0.61	13.75	**7.31**	**0.11**	**2.48**	10.44	0.25	5.68	14.10	0.46	10.46
2003	2012	5.38	0.95	13.73	0.18	0.89	12.95	**7.47**	**0.12**	**1.71**	8.63	0.44	6.44	9.33	0.80	11.65
2003	2013	6.46	1.27	12.92	0.22	1.19	12.14	**7.39**	**0.12**	**1.21**	7.01	0.74	7.56	6.23	1.25	12.69
2003	2014	7.49	1.61	12.58	0.25	1.51	11.80	**7.42**	**0.12**	**0.91**	6.22	0.91	7.08	4.93	1.52	11.85
2003	2015	8.31	1.83	11.64	0.28	1.71	10.83	**7.03**	**0.42**	**2.64**	5.01	1.53	9.74	3.36	2.37	15.02
2003	2016	9.45	2.42	11.11	0.31	2.26	10.37	**7.05**	**0.40**	**1.84**	4.46	1.75	8.04	2.65	2.78	12.75
2003	2017	11.91	5.08	13.94	0.39	4.92	13.51	8.49	3.11	8.52	**5.24**	**2.94**	**8.06**	3.09	3.38	9.27
2003	2018	14.33	7.07	15.59	0.47	6.88	15.15	9.68	4.30	9.49	**5.73**	**3.39**	**7.47**	3.24	3.40	7.48
2003	2019	16.71	9.10	16.85	0.55	8.87	16.85	10.89	5.89	10.91	6.32	4.58	8.48	**3.52**	**4.18**	**7.73**

for each row.

For Facebook, Metcalfe's law has the smallest fitting errors for periods 2004–2010, 2004–2011, 2004–2012, and 2004–2013, and the Cube law has the smallest fitting errors for more recent periods.

For Tencent, we see a similar transition from Metcalfe's law to the Cube law, except that the transition year is 2017. That is, Metcalfe's law has the smallest fitting errors for period 2003–2010 to period 2003–2016. In the 2003–2019 period, the RMSE and the NRMSE of the Quad law are smaller than those of Metcalfe's law and the Cube law.

Comparing the transition years of Facebook and Tencent, we find that the transition to the Cube law happened later for Tencent (in 2017) than for Facebook (in 2014), while Tencent transitioned to the Quad law earlier (in 2019). We provide a possible reason that the Facebook network is more open than the Tencent network, and it is easier for users to establish sub-groups. Therefore Facebook can transition to the Cube law earlier. Tencent's MAUs decreased from 1.91 billion in 2018 to 1.89 billion in 2019, while the revenue increased from \$45.46 billion in 2018 to \$54.08 billion in 2019. The reverse trend change of the MAUs and the revenue led to the best fitting result of the Quad law for Tencent in 2019. There are two possible reasons for the reverse trend change: one is that the value created by each user increases, and the other is that the monetization ratio is not constant.

Tencent's revenue[⑤] is more diversified, which may explain Tencent's continued revenue growth after the slowdown in MAUs growth. Up to 97.90% of Facebook's revenue came from advertising in 2020. For Tencent, Value-added Services, FinTech and Business Services, Online Advertising, and Others took up 53.01%, 26.86%, 18.12%, and 2.01% of Tencent's revenue respectively.

As Van Hove has done in 2015[10], we filter out revenues (including e-Commerce transactions, FinTech and Business Services, and Others) that are unrelated to Tencent's social network services. The filtered revenues are shown in the Appendix A1 of the supplementary material of the paper[⑥] and the network value functions and their RMSEs and NRMSEs are shown in Table 6. After filtering, Tencent's transition year is delayed to 2018. Metcalfe's law has the smallest fitting errors for period 2003–2010 to period 2003–2017. In the 2003–2018 and 2003–2019 periods, the Cube functions show the best fitting results.

Comparing the coefficients (a in the functions) of Facebook and Tencent, we can find that Tencent's coefficients are bigger than Facebook's before filtering. After filtering, the coefficients of the networks are similar. It may indicate that Facebook and Tencent users provide similar value on social network services, while Tencent gains higher value through more diverse services.

The coefficients, RMSEs, and NRMSEs of the network value functions of 6-year rolling periods of Facebook and Tencent are publicly available in the supplementary material of the paper[⑥] and are labeled as Appendix A2 and Appendix A3 respectively. Appendix A4 shows the network functions with the filtered revenues of Tencent. We observe similar transitions from Metcalfe's law to the Cube law. The transition years of Facebook and Tencent are 2014 and 2017, which is consistent with the conclusions in Table 4 and Table 5 respectively.

In 2015, Van Hove proposed a "network quality" indicator to present the quality of the companies' ser-

[⑤]Tencent financial reports. http://www.tencent.com/en-us/ir/reports.shtml, Nov. 2022.
[⑥]https://github.com/zxzStar/Network-effect-laws, Nov. 2022.

Table 6. Tencent's Coefficients (a), RMSEs, and NRMSEs of the Network Value Functions (Filtering out Revenues Unrelated to Social Network Services)

Start Year	End Year	Sarnoff's Function a	Sarnoff's Function RMSE	Sarnoff's Function NRMSE (%)	Odlyzko's Function a	Odlyzko's Function RMSE	Odlyzko's Function NRMSE (%)	Metcalfe's Function $a\times 10^{-9}$	Metcalfe's Function RMSE	Metcalfe's Function NRMSE (%)	Cube Function $a\times 10^{-18}$	Cube Function RMSE	Cube Function NRMSE (%)	Quad Function $a\times 10^{-27}$	Quad Function RMSE	Quad Function NRMSE (%)
2003	2010	3.41	0.43	14.98	0.12	0.41	14.14	**6.92**	**0.07**	**2.41**	11.63	0.21	7.20	18.47	0.35	12.18
2003	2011	4.28	0.64	14.46	0.15	0.61	13.67	**7.27**	**0.11**	**2.41**	10.38	0.25	5.67	14.03	0.46	10.45
2003	2012	5.10	0.82	11.88	0.17	0.77	11.14	**7.02**	**0.13**	**1.92**	8.04	0.54	7.84	8.64	0.88	12.72
2003	2013	5.77	0.96	9.75	0.20	0.89	9.04	**6.49**	**0.29**	**2.93**	6.08	0.90	9.18	5.35	1.34	13.62
2003	2014	6.73	1.35	10.53	0.23	1.26	9.84	**6.62**	**0.29**	**2.29**	5.53	0.95	7.40	4.37	1.46	11.40
2003	2015	7.63	1.69	10.73	0.26	1.58	10.00	**6.46**	**0.32**	**2.06**	4.63	1.31	8.32	3.11	2.05	13.02
2003	2016	8.58	2.12	9.74	0.29	1.98	9.08	**6.38**	**0.33**	**1.53**	4.03	1.61	7.41	2.40	2.55	11.69
2003	2017	10.40	3.93	10.77	0.34	3.78	10.37	**7.36**	**2.13**	**5.84**	4.53	2.18	5.99	2.66	2.75	7.55
2003	2018	11.92	4.97	10.94	0.39	4.79	10.55	7.97	2.58	5.69	**4.68**	**2.19**	**4.82**	2.63	2.67	5.88
2003	2019	13.33	5.98	11.07	0.44	5.78	10.71	8.59	3.30	6.11	**4.94**	**2.56**	**4.74**	2.74	2.77	5.13

Note: the unit of RMSE is billion USD.

vices, which can be calculated by cost per MAU[10]. This paper follows Van Hove's methodology to define the network value functions with quality:

$$V = a \times SIZE + q \times QUALITY,$$

while $SIZE$ equals n, $n\log(n)$, n^2, n^3, and n^4 respectively, and $QUALITY$ stands for "network quality". Appendix A5 and Appendix A6 in the supplementary material⑦ present the fitting results of the network value functions with quality of 6-year rolling periods. For all network value functions, the RMSEs and NRMSEs are lower when adding the quality indicators. However, Sarnoff's function, Odlyzko's function, and Metcalfe's function are collapsed since some coefficients present negative signs.

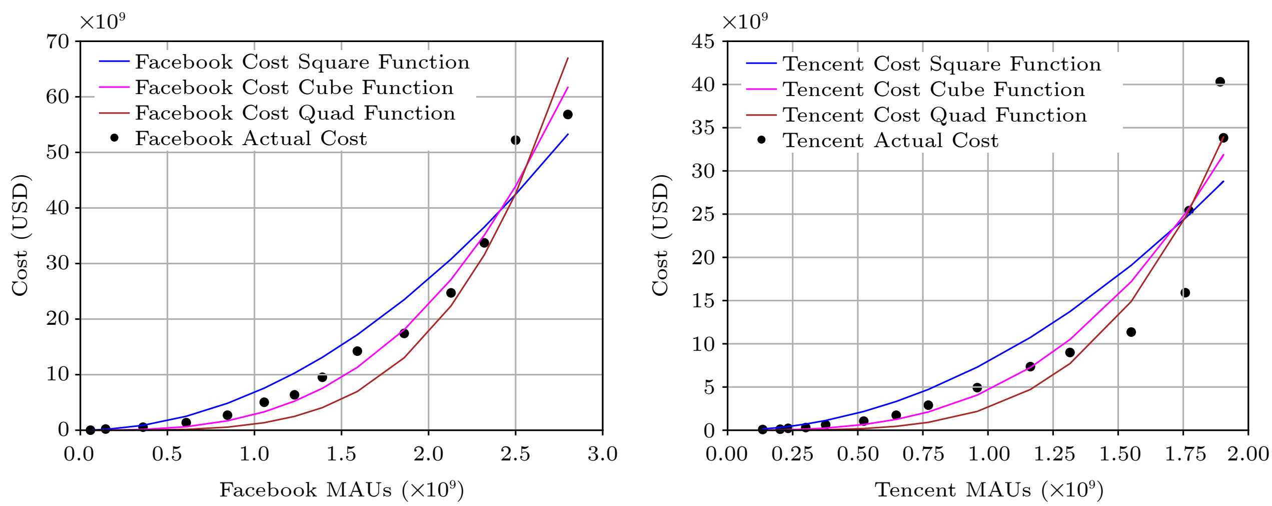
4.2 Cost Functions

By using the actual data to fit the cost functions, we find that the relationship between the cost and the size also fits the Cube function.

The fitting results of Facebook and Tencent actual data are shown in Fig.2 and Table 7. For Facebook and Tencent, the NRMSE of the Cost Cube function is the smallest among the cost functions, which is consistent with the value functions. The NRMSEs of the Cost Cube function for Facebook and Tencent are 5.14% and 6.86%, respectively.

4.3 Trend Functions

Metcalfe proposed the netoid function[5], a particu-



(a) (b)

Fig.2. Cost curves of (a) Facebook and (b) Tencent.

⑦https://github.com/zxzStar/Network-effect-laws, Nov. 2022.

Table 7. Fitting Results of the Cost Functions

	Facebook Data (2007–2020)		Tencent Data (2003–2019)	
	Cost Function	NRMSE (%)	Cost Function	NRMSE (%)
Cost Square function	$C_{\text{Facebook}} = 6.79 \times 10^{-9} \times n^2$	7.27	$C_{\text{Tencent}} = 7.94 \times 10^{-9} \times n^2$	11.80
Cost Cube function	$C_{\text{Facebook}} = 2.81 \times 10^{-18} \times n^3$	**5.14**	$C_{\text{Tencent}} = 4.61 \times 10^{-18} \times n^3$	**6.86**
Cost Quad function	$C_{\text{Facebook}} = 1.09 \times 10^{-27} \times n^4$	8.70	$C_{\text{Tencent}} = 2.58 \times 10^{-27} \times n^4$	7.97

lar S-curve adoption function, to estimate the growth trend of network size n with respect to time t. Fig.3 presents the growth trend of MAUs of Facebook and Tencent. The netoid functions are as follows:

$$Netoid_{\text{Facebook}} = \frac{3.03 \times 10^9}{1 + \mathrm{e}^{-0.35\times(t-2014.40)}},$$

$$Netoid_{\text{Tencent}} = \frac{2.18 \times 10^9}{1 + \mathrm{e}^{-0.34\times(t-2012.56)}}.$$

The NRMSEs of Facebook and Tencent are 3.56% and 2.00% respectively, which show that the growth trend satisfies the netoid function well.

It is interesting to compare these results with those of the 2015 study[6], where the netoid functions for Facebook and Tencent are as follows:

$$Netoid_{\text{Facebook}} = \frac{1.45 \times 10^9}{1 + \mathrm{e}^{-0.77\times(t-2010.56)}},$$

$$Netoid_{\text{Tencent}} = \frac{2.61 \times 10^9}{1 + \mathrm{e}^{-0.30\times(t-2013.8)}}.$$

Note that the specific parameter values have changed. Facebook's user growth speed decreases from 0.77 billion MAUs per year in 2014 to 0.35 billion MAUs per year in 2020. Tencent's user growth speed remains almost unchanged, from 0.30 billion MAUs per year in 2014 to 0.34 billion MAUs per year in 2019. The predicted peak number of MAUs of Facebook increases from 1.45 billion to 3.03 billion, while that of Tencent decreases from 2.61 billion to 2.18 billion.

5 Discussions

Although Facebook and Tencent have big differences in revenue, cost, business model, and technology, both of their actual data fit the Cube law better than Metcalfe's law. Why?

We offer a possible explanation. The network value is affected not only by the number of connections, but also by the number of sub-groups of the social network, as postulated by Reed[9]. Metcalfe's law claims that the value of the network is proportional to the number of the connections, and the number of potential connections in a network of n nodes is $n(n-1)$, which is asymptotically n^2. However, according to Reed, the value of a large social network can scale exponentially with the size of the network, which is based on the fact that the number of possible sub-groups is 2^n. A law depicting the value of future large social networks should lie somewhere between Metcalfe's law ($V \propto n^2$) and Reed's law ($V \propto 2^n$).

What trends will the network value show in the future for large social networks?

For the next decade, the MAUs as well as the values of large social networks may continue to grow. However, the growth speed of the network size may decrease under the combined effect of the following four factors.

1) *Natural Limit*. The size of large social networks is unlikely to grow indefinitely. The total population of the world, which is 7.59 billion in 2020, may

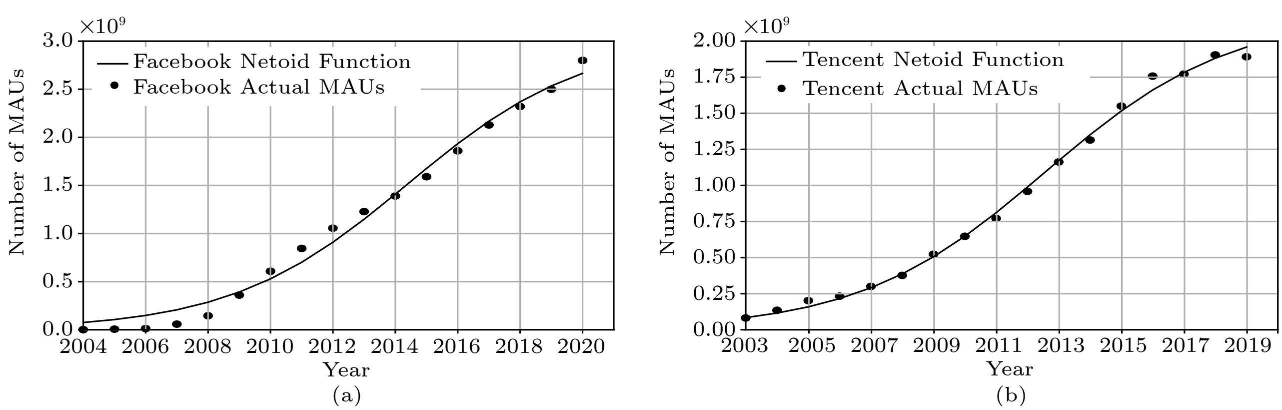


Fig.3. Netoid curves of (a) Facebook and (b) Tencent.

impact the natural limits of MAUs.

2) *Decentralization*. Scholars are calling for decentralized technologies to change the current Web ecosystem to protect the ownership and privacy of users[16], which may reduce user stickiness and thus reduce the MAUs of large social networks. For example, the inventor of the World Wide Web, Tim Berners-Lee, launched a web decentralization project called Solid[17] (for Social Online Data) to enable users to have full control of their online data.

3) *Anti-Monopoly*. Governments around the world are conducting anti-monopoly investigations, which will increase the number of competitors of large social network companies[18, 19].

4) *Objective Factor*. Objective factors will affect the MAUs and revenue of social networks, resulting in deviations in data fitting of network effects. For example, according to Facebook's financial report⑧, due to the impact of the COVID-19 epidemic, Facebook's MAUs have grown significantly, while advertising demand and prices have fallen since the first quarter of 2020.

Even when the growth in MAUs slows, the network value may continue to grow if the value per user grows. Facebook's per-user value increased from 8.97 dollars in 2014 to 30.70 dollars in 2020. Tencent's per-user value increased from 9.81 dollars in 2014 to 29.85 dollars in 2019. In 2014, due to low access bandwidth and unreliable/unstable data connection, the quality of Internet connections in China was low[20], and since then, as the quality of Internet connections has improved, it may lead to an increase in per-user value. However, the cost per user may also grow. Facebook's per-user cost increased from 6.86 dollars in 2014 to 20.30 dollars in 2020. Tencent's per-user cost increased from 6.84 dollars in 2014 to 21.31 dollars in 2019.

Do Metcalfe's law and the Cube law only work for Tencent and Facebook? This paper leverages the data of more social network companies, Weibo and LinkedIn, to fit the network effect laws. Appendix A7 in the supplementary material⑨ presents the Actual Data of Weibo and LinkedIn. Appendix A8 and Appendix A9 present the coefficients, RMSEs, and NRMSEs of the network value functions with 4-year rolling periods in the supplementary material. For Weibo, Metcalfe's law has the smallest fitting errors for period 2013–2017 to period 2017–2021, and Odlyzko's law has the smallest fitting errors for period 2012–2016. For LinkedIn, Odlyzko's law has the smallest fitting errors for period 2010–2014 to period 2013–2017, and Metcalfe's law has the smallest fitting errors for more recent periods. By fitting these social network data, Metcalfe's law shows good generality. Because the user scale of Weibo and LinkedIn is smaller than that of Facebook and Tencent, the transition from Metcalfe's law to the Cube law has not been observed.

6 Conclusions

Facebook's actual data over 2004–2020 and Tencent's actual data over 2003–2019 show that the value of a social network fits a Cube law better than Metcalfe's law. That is, the value of a social network is proportional to the cube of the number of users n. The value functions of these two companies currently stand at

$$V_{\text{Facebook}} = 4.20 \times 10^{-18} \times n^3,\ \ RMSE = 2.19\ \times 10^9 \text{ USD},$$

and

$$V_{\text{Tencent}} = 6.32 \times 10^{-18} \times n^3,\ \ RMSE = 4.58\ \times 10^9 \text{ USD},$$

respectively.

The actual data still fit Metcalfe's law. The Metcalfe's functions stand at

$$V_{\text{M-Facebook}} = 10.13 \times 10^{-9} \times n^2,\ \ RMSE = 4.86\ \times 10^9 \text{ USD},$$

and

$$V_{\text{M-Tencent}} = 10.9 \times 10^{-9} \times n^2,\ \ RMSE = 5.89\ \times 10^9 \text{ USD},$$

respectively.

However, the fitting errors of Facebook and Tencent, in terms of normalized root-mean-square errors (NRMSEs), are smaller for the Cube law (2.55% and 8.48%) than for Metcalfe's law (5.65% and 10.91%).

Real data also show that the cost of a network fits the Cube law, instead of a linear law. The cost functions of Facebook and Tencent are

$$C_{\text{Facebook}} = 2.81 \times 10^{-18} \times n^3,$$

$$C_{\text{Tencent}} = 4.61 \times 10^{-18} \times n^3,$$

respectively. The NRMSEs of the Cost Cube functions for Facebook and Tencent are 5.14% and 6.86% respectively.

The growth trend of MAUs of Facebook and Ten-

⑧Facebook financial reports. http://investor.fb.com/, Nov. 2022.
⑨https://github.com/zxzStar/Network-effect-laws, Nov. 2022.

cent can be modeled by the following netoid functions:

$$Netoid_{\text{Facebook}} = \frac{3.03 \times 10^9}{1 + \mathrm{e}^{-0.35 \times (t - 2014.40)}},$$

$$Netoid_{\text{Tencent}} = \frac{2.18 \times 10^9}{1 + \mathrm{e}^{-0.34 \times (t - 2012.56)}},$$

respectively.

The NRMSEs of Facebook and Tencent are 3.56% and 2.00% respectively.

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